A SYSTEMATIC WAY TO GRAPH POLAR FUNCTIONS

To graph $r = f(\theta)$:

1. Run symmetry tests to determine the minimum interval of the graph you need to plot first.
2. Solve $f(\theta) = 0$ to determine the angles in the minimum interval at which the graph goes through the pole. At those points, the graph will also be tangent to the line through the pole at that angle.
3. Find the values of $f(\theta)$ for all common angles in the minimum interval from step 1.
4. Plot the tangent lines and points in steps 2 and 3, and connect them in counterclockwise order.
5. Reflect the graph in step 4 into the other quadrants according to the results of the symmetry tests.

SYMMETRY TESTS

Symmetry of graph Polar terminology	**Symmetry of graph Rectangular terminology**	**Tests**	**Minimum Interval**
over the polar axis	over the x-axis	$(r, -\theta)$ $(-r, \pi-\theta)$	$\theta \in [0, \pi]$
over $\theta = \frac{\pi}{2}$	over the y-axis	$(r, \pi-\theta)$ $(-r, -\theta)$	$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
about the pole	through the origin	$(r, \pi+\theta)$ $(-r, \theta)$	either $\theta \in [0, \pi]$ or $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Notes regarding symmetry tests:

A. Run each symmetry test by replacing (r, θ) in the function with the corresponding ordered pair above.
If the equation can then be simplified to the original function, then the graph has that type of symmetry.
If the equation cannot be simplified to the original function, then the test has no conclusion.
The result of a symmetry test is NEVER "the graph is not symmetric".

B. Each symmetry test actually has infinitely many variations, only two of which are listed above.
The other variations result from replacing the angle in the test with every co-terminal angle.
For this class, we will only run the symmetry tests above.

C. If a function passes one version of a symmetry test, it is not necessary to run the other version.

D. If a function has two of the three types of symmetry, it automatically has all three types.
In that case, it not necessary to run the third set of symmetry tests,
and the minimum interval becomes $\theta \in \left[0, \frac{\pi}{2}\right]$.

BEWARE: If a graph has one type of symmetry, but does not pass either test for a second type of symmetry, **you must still test for the third type of symmetry**, since the result of the second symmetry test is "no conclusion", NOT "the graph is not symmetric".

Example: Graph $r = \sin 2\theta$

1. Symmetry over polar axis

$$r = \sin 2(-\theta)$$
$$r = -\sin 2\theta$$
NO CONCLUSION

$$-r = \sin 2(\pi - \theta)$$
$$-r = \sin(2\pi - 2\theta)$$
$$-r = \sin 2\pi \cos 2\theta - \cos 2\pi \sin 2\theta$$
$$-r = -\sin 2\theta$$
$$r = \sin 2\theta$$
SYMMETRIC OVER POLAR AXIS

Symmetry about pole

$$r = \sin 2(\pi + \theta)$$
$$r = \sin(2\pi + 2\theta)$$
$$r = \sin 2\pi \cos 2\theta + \cos 2\pi \sin 2\theta$$
$$r = \sin 2\theta$$
SYMMETRIC ABOUT POLE – skip other test

Since the graph has these two types of symmetry, it automatically has the third type of symmetry ie. it is also symmetric over $\theta = \frac{\pi}{2}$.

So, the minimum interval we need to plot first is $\theta \in \left[0, \frac{\pi}{2}\right]$.

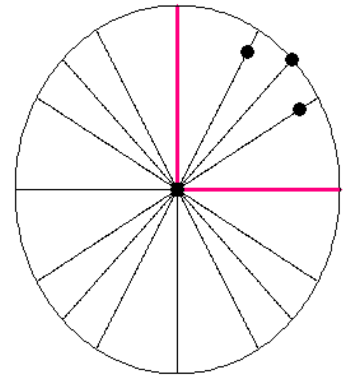
2. $\sin 2\theta = 0$ for $\theta \in \left[0, \frac{\pi}{2}\right]$ when $\theta = 0, \frac{\pi}{2}$

The graph will pass through the pole when $\theta = 0, \frac{\pi}{2}$ and will be tangent to the lines corresponding to $\theta = 0, \frac{\pi}{2}$

3. The common angles in the minimum interval $\left[0, \frac{\pi}{2}\right]$ are $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

θ	$r = \sin 2\theta$
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2} \approx 0.9$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.9$
$\frac{\pi}{2}$	0

4. Plotting the two tangent lines from step 2
and the five points from step 3 (two of which are at the pole) gives



Starting at $\theta = 0$ [the polar point $(0, 0)$],

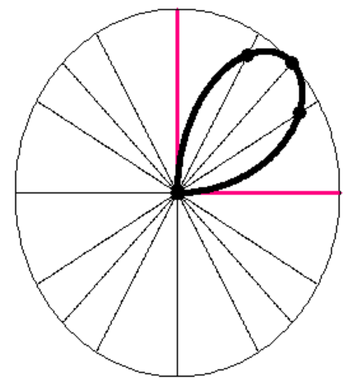
connect counterclockwise to $\theta = \frac{\pi}{6}$ [the polar point $\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$],

then counterclockwise to $\theta = \frac{\pi}{4}$ [the polar point $\left(1, \frac{\pi}{4}\right)$],

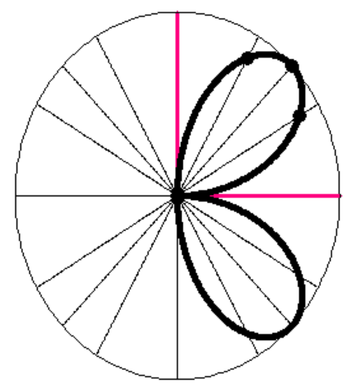
then counterclockwise to $\theta = \frac{\pi}{3}$ [the polar point $\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right)$],

and finally to $\theta = \frac{\pi}{2}$ [the polar point $\left(0, \frac{\pi}{2}\right)$, ie. the pole again].

Your curve should be tangent to the tangent lines you drew before.

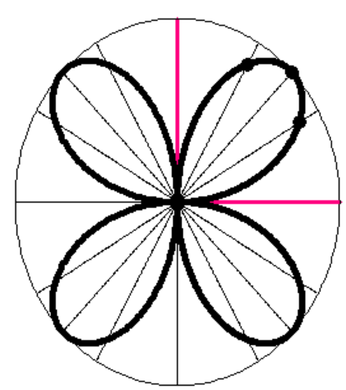


5. Since the graph is symmetric over the polar axis,
reflect the partial graph vertically.



Since the graph is also symmetric over $\theta = \frac{\pi}{2}$,

reflect the new partial graph horizontally.



The graph is now complete. (You can unhighlight the points and tangent lines from step 4.)